What is the q-analogue of a graph?

Relinde Jurrius

Netherlands Defence Academy

Open problem session on graphs & matroids
December 14, 2020

What is the q-analogue of a graph?

Its cycles should be the circuits of a q-matroid:

q-matroid: a pair $(E, \mathcal{C})$ with

- $E = \mathbb{F}_q^n$ finite dimensional vector space;
- $\mathcal{C}$ family of subspaces of E, the *circuits*, with
  - (C1) $0 \notin \mathcal{C}$.
  - (C2) If $C_1, C_2 \in \mathcal{C}$ and $C_1 \subseteq C_2$, then $C_1 = C_2$.
  - (C3) For all $C_1, C_2 \in \mathcal{C}$ distinct and $X \subseteq E$ of codimension 1, there is a $C_3 \in \mathcal{C}$ such that $C_3 \subseteq (C_1 + C_2) \cap X$.

What is the q-analogue of a graph?

Wild speculation:

graph	$\rightsquigarrow$	hypergraph
$k+1$ vertices	$\rightsquigarrow$	all 1-dim spaces of $\mathbb{F}_q^{k+1}$ i.e., points in $\mathrm{PG}(k,q)$
n edges	$\rightsquigarrow$	hyperedges coming from 2-dim spaces i.e., lines in $\mathrm{PG}(k,q)$
cycles	$\rightsquigarrow$	???